PERIODIC ACTIONS ON DISTRIBUTIVE LATTICES AND COUNTERPARTS IN ALGEBRA

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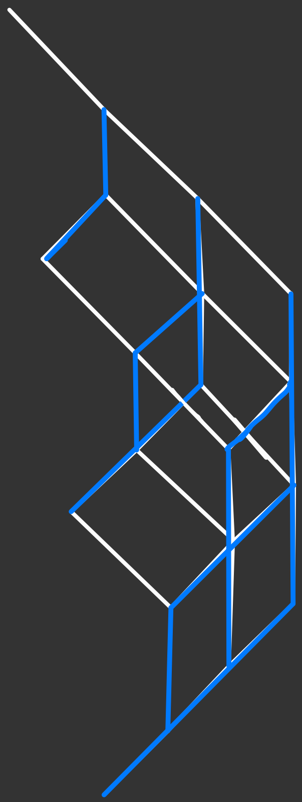
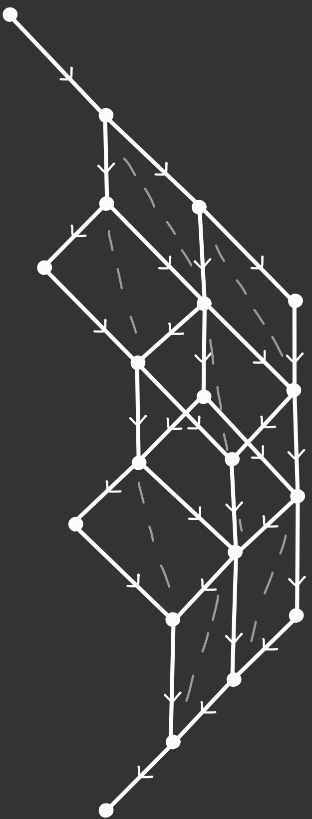


Let L be a distributive lattice.

A be the incidence algebra of L.



THEOREM: (Iyama - Marczinzik)

Lattices are Auslander regular if and only if they are distributive.

- A finite dimensional algebra A is called Auslander-regular if A has finite global dimension and in the minimal injective coresolution

$$0 \to A \to I_0 \to \cdots \to I_n \to 0$$

we have that projective dimension of I_i is bounded by i for all $i \geq 0$.

Example: For an n-representation finite algebra Λ with n-cluster tilting module M, the endomorphism algebra $B := \mathrm{End}_\Lambda(M)$ will be an higher Auslander algebra that is Auslander regular.

Let Λ be an Auslander regular algebra.

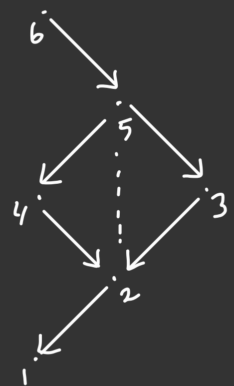
Grade bijection :

$$d = \operatorname{grade} M := \inf\{ i \geq 0 \mid \operatorname{Ext}^i_\Lambda(M,\Lambda) \neq 0 \}$$

$$S \longmapsto \operatorname{top}(D\operatorname{Ext}^d_\Lambda(S,\Lambda))$$

simple
module

Example:



* $S_2 \longmapsto \mathrm{top}\left(D\,\mathrm{Ext}^1(S_2, A)\right)$

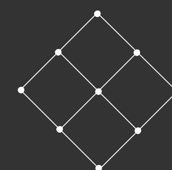
$$0 \to P_1 \to P_2 \to S_2 \to 0$$

$$0 \to \mathrm{Hom}(S_2, A) \to \mathrm{Hom}(P_2, A) \to \mathrm{Hom}(P_1, A) \to \mathrm{Ext}^1(S_2, A) \to 0$$

$$D\,\mathrm{Ext}^1(S_2, A) = S_1$$

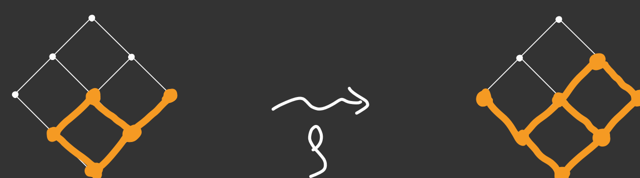
* $S_5 \longmapsto S_2$

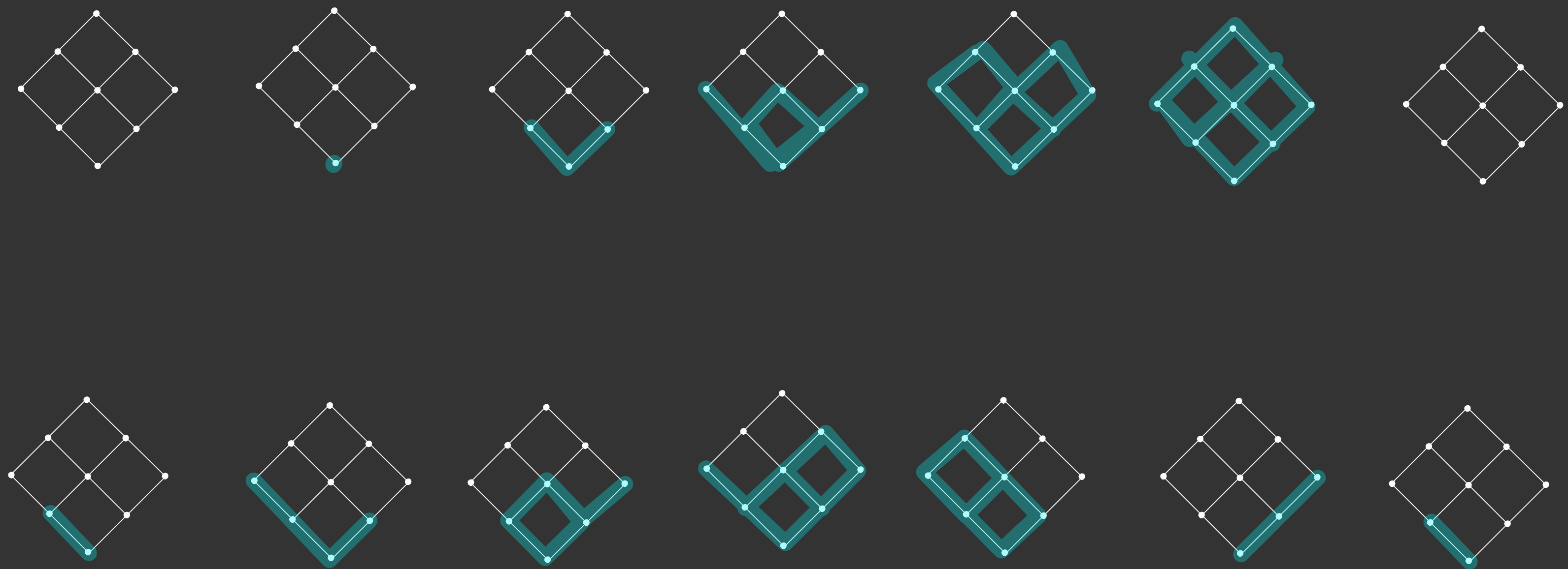
Grade bijection coincides with a well-known action "Rowmotion" for $\mathcal{A}$ (the incidence algebra of L)

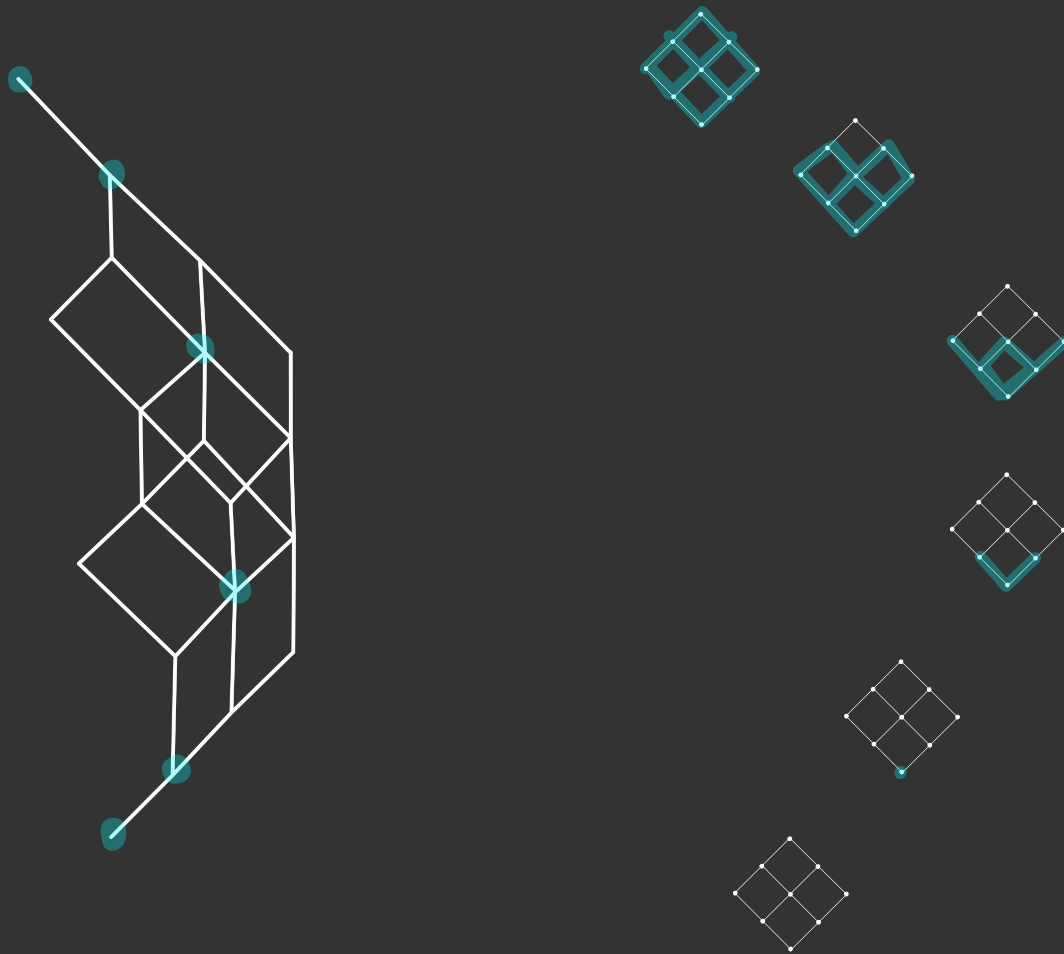
Example. Take a poset as 

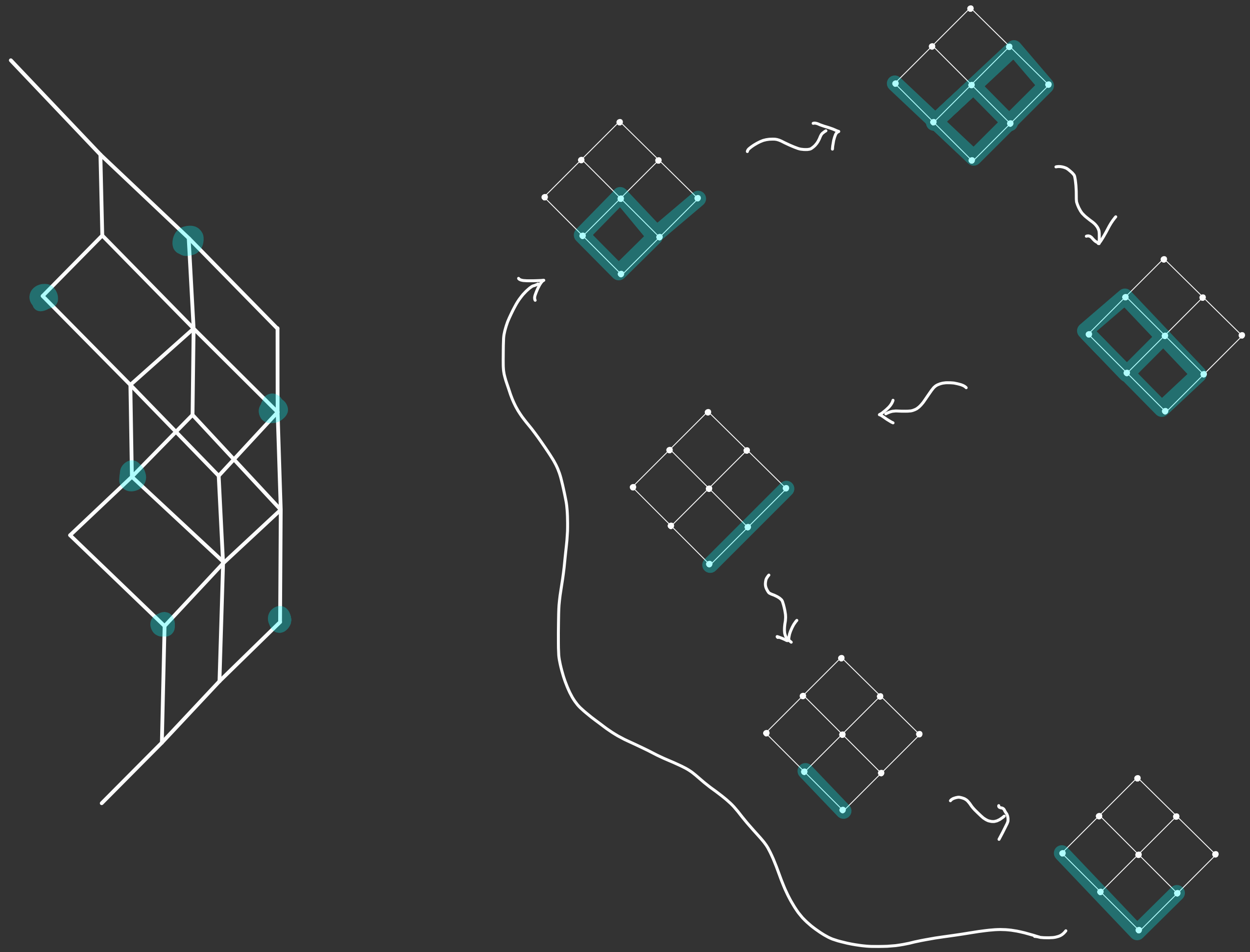
Take an order ideal I (dowclosed subset)  I

- The rowmotion on I, $\rho(I)$ is the order ideal generated by the minimal elements of P not in I. (see Striker - Williams)









THEOREM (Marczinzik - Thomas - Y)

Let A be the incidence algebra of $\mathcal{L}$, then

$(\rho\tau)^2 = id$ where τ is the Coxeter transformation.

$$(\rho\tau)^2 = \mathrm{id}$$

$$0 \to S_5 \to 0$$

$$0 \to P_2 \to P_5 \to 0$$

$$0 \to I_2 \to I_5 \to 0 \qquad \tau$$

$$0 \to {}_{2}{}^{3}{}^{4} \to 0$$

$$\rho\left({}_{2}{}^{3}{}^{4}\right) = {}_{5}{}^{6}{}^{7}$$

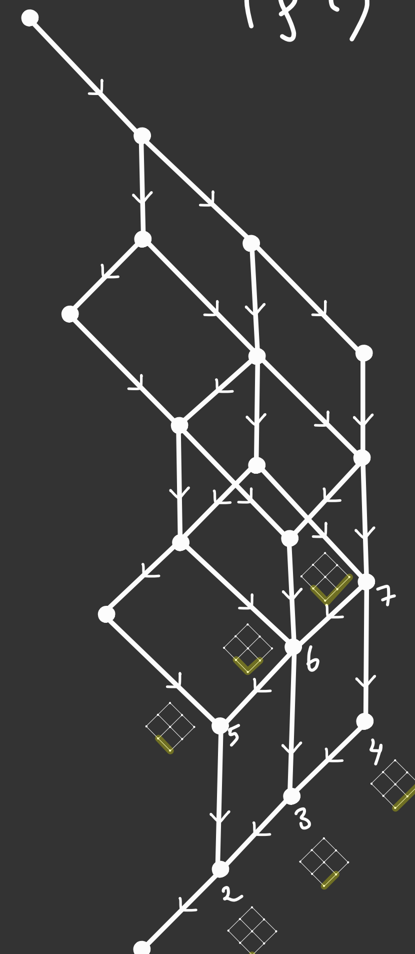
$$0 \to {}_{5}{}^{6}{}^{7} \to 0$$

$$0 \to P_4 \to P_7 \to 0$$

$$0 \to I_4 \to I_7 \to 0 \qquad \tau$$

$$0 \to S_4 \to 0$$

$$\rho(S_4) = S_5$$



THEOREM: (MTY)

- Let Λ be an n-representation finite algebra with n-cluster tilting module M, for the endomorphism algebra $B := \mathrm{End}_\Lambda(M)$ we have

$$(\rho C)^2 = \mathrm{id} \quad \text{if } n \text{ is even,}$$

$$(\rho C + \mathrm{id})^2 = 0 \quad \text{if } n \text{ is odd}$$