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Higher Koszul duality and connections with *n*-hereditary algebras

Johanne Haugland

Joint work with Mads H. Sandøy

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Introduction

Joint with Mads H. Sandøy: *Higher Koszul duality and connections with n-hereditary algebras* (2021, arXiv:2101.12743)

Introduction

Koszul duality

Higher homological algebra

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Koszul duality ~ **Higher homological algebra**

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***n*-hereditary algebras**

- Iyama–Oppermann ’11 and ’13
- Herschend–Iyama–Oppermann ’14

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Generalized Koszul algebras

- Green–Reiten–Solberg ’02
- Madsen ’11

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} T-Koszul

Conventions and notation

1. $n =$ positive integer
2. $k =$ algebraically closed field
3. All algebras are algebras over k
4. $D(-) = \mathrm{Hom}_k(-, k)$
5. A and $B =$ ungraded algebras
6. $\Lambda =$ positively graded algebra
7. $\mathrm{mod}\, A =$ finitely presented right A-modules
8. $\mathrm{gr}\, \Lambda =$ finitely presented graded right Λ-modules

n-hereditary algebras

Classical case ($n = 1$)

n-hereditary algebras

Classical case ($n = 1$)

hereditary alg. A — $\{$ gldim $A \leq 1$, $A \simeq kQ$ $\}$

rep. finite

rep. infinite

n-hereditary algebras

Let A be a finite dimensional algebra with $\operatorname{gl.dim} A \leq n$.

Nakayama functor

$\nu = D\,\mathrm{RHom}_A(-, A)\colon\ \mathcal{D}^b(\operatorname{mod} A) \xrightarrow{\simeq} \mathcal{D}^b(\operatorname{mod} A)$
$\nu^{-1} = \mathrm{RHom}_A(DA, -)\colon\ \mathcal{D}^b(\operatorname{mod} A) \xrightarrow{\simeq} \mathcal{D}^b(\operatorname{mod} A)$

We use the notation $\nu_n = \nu \circ [-n]$.

Auslander–Reiten translation

For $n = 1$, we have $\tau \simeq \mathrm{H}^0(\nu_1)\colon\ \operatorname{mod} A \to \operatorname{mod} A$

Higher Auslander–Reiten translation

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$= \tau \Omega^{n-1}$

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n-hereditary algebras

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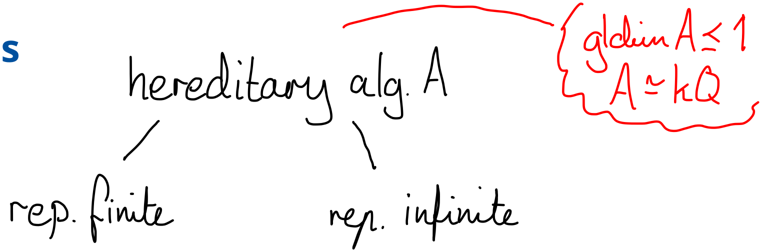

n-hereditary algebras

Classical case ($n = 1$)

hereditary alg. A — $\{\operatorname{gldim} A \leq 1,\ A \simeq kQ\}$

- rep. finite
- rep. infinite

rep. finite $\Longleftrightarrow$ $\operatorname{mod} A = \operatorname{add}\{\tau^{-i}A \mid i \geq 0\}$

n-hereditary algebras

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Check: $M \in \operatorname{mod} A$ indec. non-injective $\implies$ $\nu_1^{-1}M$ cons. in degree 0

n-hereditary algebras

Let A be a finite dimensional algebra with $\operatorname{gl.dim} A \leq n$.

Definition

1. A is called *n-representation finite* if there for each indecomposable $P \in \operatorname{proj} A$ exists an integer $i \geq 0$ such that $\nu_n^{-i} P$ is indecomposable injective.
2. A is called *n-representation infinite* if $\mathrm{H}^i(\nu_n^{-j} A) = 0$ for $i \neq 0$ and $j \geq 0$.
3. A is called *n-hereditary* if it is either n-representation finite or n-representation infinite.

In this talk:

All n-hereditary algebras are assumed to be basic.

n-hereditary algebras

Classes of examples of n-representation finite algebras

1. Higher type A algebras [Iyama–Oppermann ’11]
2. Tensor products of ℓ-homogeneous higher representation finite algebras [Herschend–Iyama ’11]
3. Nakayama algebras with homogeneous relations [Darpö–Iyama ’20, Vaso ’19]

n-hereditary algebras

Classes of examples of n-representation infinite algebras

1. Higher type $\widetilde{A}$ algebras [Herschend–Iyama–Oppermann ’14]
2. Tensor products of higher representation-infinite algebras [Herschend–Iyama–Oppermann ’14]

n-hereditary algebras

Higher preprojective algebras

Given an n-hereditary algebra A, the $(n+1)$-*preprojective algebra* of A is given by

$$\Pi_{n+1}A = \bigoplus_{i \geq 0} \mathrm{Hom}_{\mathcal{D}^b(A)}(A, \nu_n^{-i} A).$$

Koszul algebras

A graded algebra $\Lambda = \oplus_{i\geq 0}\Lambda_i$ which is generated in degrees 0 and 1 with Λ_0 semisimple is known as a *Koszul algebra* if

$$\operatorname{Ext}^i_{\operatorname{gr}\Lambda}(\Lambda_0, \Lambda_0\langle j\rangle) = 0$$

for $i \neq j$.

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The *Koszul dual* of Λ is defined as

$$\Lambda^! = \bigoplus_{i\geq 0} \operatorname{Ext}^i_{\operatorname{gr}\Lambda}(\Lambda_0, \Lambda_0\langle i\rangle).$$

Koszul duality

Let Λ be a Koszul algebra and $\Lambda^!$ its Koszul dual. Given certain finiteness conditions, we have

$$\mathcal{D}^b(\operatorname{gr}\Lambda) \xrightarrow{\simeq} \mathcal{D}^b(\operatorname{gr}\Lambda^!).$$

Aim

Generalize the notion of Koszul algebras and get a *higher* version of the Koszul duality equivalence above.

Trivial extensions

Let A be a finite dimensional algebra. The *trivial extension of A* is

$$\Delta A = A \oplus DA$$

with multiplication $(a, f) \cdot (b, g) = (ab, ag + fb)$ for $a, b \in A$ and $f, g \in DA$.

ΔA can be graded with A in degree 0 and DA in degree 1.

Graded symmetric algebras

A finite dimensional algebra $\Lambda = \oplus_{i \geq 0} \Lambda_i$ is *graded symmetric* if $D\Lambda \simeq \Lambda\langle -a \rangle$ as graded Λ-bimodules for some integer a.

Note

1. Any graded symmetric algebra is self-injective.
2. The integer a must be equal to the highest degree of Λ.

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Example

The trivial extension ΔA of a finite dimensional algebra A is graded symmetric.

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Example

$= A \oplus DA$

The trivial extension ΔA of a finite dimensional algebra A is graded symmetric.

$a = 1, \quad D(\Delta A) \simeq \Delta A\langle -1 \rangle$

Motivation

Finiteness condition

The category $\mathrm{gr}\,\Lambda$ is abelian if and only if Λ is *graded right coherent*, i.e. if every finitely generated homogeneous right ideal is finitely presented.

Some known equivalences

Let A be an n-representation infinite algebra with $\Pi_{n+1}A$ graded right coherent. We then have

$$\underline{\mathrm{gr}}\,\Delta A \simeq \mathcal{D}^b(\mathrm{mod}\,A) \simeq \mathcal{D}^b(\mathrm{qgr}\,\Pi_{n+1}A).$$

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Happel

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Happel

Minamoto, Mori

Koszul duality and the BGG-correspondence

Let Λ be a Koszul algebra which is graded symmetric. We have

$$
\begin{array}{ccc}
\mathcal{D}^b(\operatorname{gr}\Lambda) & \xrightarrow{\simeq} & \mathcal{D}^b(\operatorname{gr}\Lambda^!) \\
\downarrow & & \downarrow \\
\underline{\operatorname{gr}}\,\Lambda & \overset{\simeq}{\dashrightarrow} & \mathcal{D}^b(\operatorname{qgr}\Lambda^!)
\end{array}
$$

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Koszul duality

BGG-correspondence

Motivating question

Is the equivalence $\underline{\operatorname{gr}}\,\Delta A \simeq \mathcal{D}^b(\operatorname{qgr}\Pi_{n+1}A)$ a consequence of some higher Koszul duality?

Generalized Koszul algebras

Tilting modules

Let A be a finite dimensional algebra. A finitely generated A-module T is called a *tilting module* if the following conditions hold:

1. $\operatorname{proj.dim}_A T < \infty$;
2. $\operatorname{Ext}^i_A(T, T) = 0$ for $i > 0$;
3. There is an exact sequence

$$0 \to A \to T^0 \to T^1 \to \cdots \to T^t \to 0$$

with $T^i \in \operatorname{add} T$ for $i = 0, \ldots, t$.

Generalized Koszul algebras

Let $\Lambda = \oplus_{i \geq 0} \Lambda_i$ be a positively graded algebra.

Definition

Let T be a finitely generated basic graded Λ-module concentrated in degree 0. We say that T is *graded n-self-orthogonal* if

$$\operatorname{Ext}^i_{\operatorname{gr}\Lambda}(T, T\langle j \rangle) = 0$$

for $i \neq nj$.

Generalized Koszul algebras

Definition

Assume $\operatorname{gl.dim} \Lambda_0 < \infty$ and let T be a graded Λ-module concentrated in degree 0. We say that Λ is *n-T-Koszul* or *n-Koszul with respect to T* if the following conditions hold:

1. T is a tilting Λ_0-module.
2. T is graded n-self-orthogonal as a Λ-module.

Generalized Koszul algebras

Definition

Assume $\text{gl.dim}\,\Lambda_0 < \infty$ and let T be a graded Λ-module concentrated in degree 0. We say that Λ is *n-T-Koszul* or *n-Koszul with respect to T* if the following conditions hold:

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2. T is graded n-self-orthogonal as a Λ-module.

Definition

Let Λ be an n-T-Koszul algebra. The *n-T-Koszul dual of* Λ is given by

$$\Lambda^! = \oplus_{i \geq 0} \operatorname{Ext}^{ni}_{\operatorname{gr}\Lambda}(T, T\langle i \rangle).$$

Generalized Koszul algebras

Example

Generalized Koszul algebras

Example

$$1 \rightrightarrows 2$$

Generalized Koszul algebras

Example

$1 \rightrightarrows 2$ — A 1-rep. infinite

Generalized Koszul algebras

Example

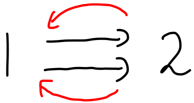
$1 \rightrightarrows 2$

A 1-rep. infinite

$\Delta A = A \oplus DA$

Generalized Koszul algebras

Example



A 1-rep. infinite

$\Delta A = A \oplus DA$

Generalized Koszul algebras

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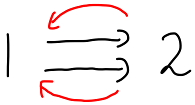


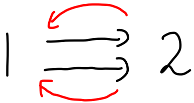
A 1-rep. infinite

$\Delta A = A \oplus DA$

Check: $\mathrm{Ext}^i_{\mathrm{gr}\,\Delta A}(A, A\langle j \rangle)$

Generalized Koszul algebras

Example



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$$\Delta A = A \oplus DA$$

Check: $\operatorname{Ext}^i_{\mathrm{gr}\,\Delta A}(A, A\langle j\rangle) = 0$ for $i \neq 2j$

Generalized Koszul algebras

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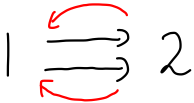

A 1-rep. infinite

$\Delta A = A \oplus DA$

Check: $\mathrm{Ext}^i_{\mathrm{gr}\text{-}\Delta A}(A, A\langle j\rangle) = 0$ for $i \neq 2j$

$\rightsquigarrow$ ΔA is 2-Koszul wrt. A

Generalized Koszul algebras

Proposition

Let A be an n-representation infinite algebra. The following statements hold:

1. The trivial extension ΔA is $(n+1)$-Koszul with respect to A.
2. We have $(\Delta A)^! \simeq \Pi_{n+1} A$ as graded algebras.

Higher Koszul duality

Theorem

Let Λ be a finite dimensional n-T-Koszul algebra and assume that $\Lambda^!$ is graded right coherent and has finite global dimension. Then there is an equivalence

$$\mathcal{D}^b(\operatorname{gr}\Lambda) \xrightarrow{\simeq} \mathcal{D}^b(\operatorname{gr}\Lambda^!)$$

of triangulated categories.

Higher Koszul duality and BGG-correspondence

Proposition

In the case where our algebra Λ is graded symmetric, the higher Koszul duality equivalence descends to yield an analogue of the BGG-correspondence

$$\begin{array}{ccc} \mathcal{D}^b(\operatorname{gr}\Lambda) & \xrightarrow{\simeq} & \mathcal{D}^b(\operatorname{gr}\Lambda^!) \\ \downarrow & & \downarrow \\ \underline{\operatorname{gr}}\,\Lambda & \overset{\simeq}{\dashrightarrow} & \mathcal{D}^b(\operatorname{qgr}\Lambda^!). \end{array}$$

Back to our motivating question

Corollary

Let A be an n-representation infinite algebra with $\Pi_{n+1}A$ graded right coherent. We then have:

$$\begin{array}{ccc} \mathcal{D}^b(\operatorname{gr} \Delta A) & \xrightarrow{\simeq} & \mathcal{D}^b(\operatorname{gr} \Pi_{n+1} A) \\ \downarrow & & \downarrow \\ \underline{\operatorname{gr}} \, \Delta A & \overset{\simeq}{\dashrightarrow} & \mathcal{D}^b(\operatorname{qgr} \Pi_{n+1} A) \end{array}$$

In particular, this holds whenever an n-representation infinite algebra A is *n-representation tame* as defined in [Herschend–Iyama–Oppermann '14].

A characterization

Tilting object

Let $\mathcal{T}$ be a triangulated category. An object T in $\mathcal{T}$ is a *tilting object* if the following conditions hold:

1. $\mathrm{Hom}_{\mathcal{T}}(T, T[i]) = 0$ for $i \neq 0$;
2. $\mathrm{Thick}_{\mathcal{T}}(T) = \mathcal{T}$.

A characterization

Notation and standing assumptions

1. $\Lambda =$ graded symmetric algebra of highest degree $a \geq 1$
2. $\operatorname{gl.dim} \Lambda_0 < \infty$
3. $T \in \operatorname{gr} \Lambda$ satisfies:
   - i) T is basic
   - ii) T is concentrated in degree 0
   - iii) T is a tilting module over Λ_0
4. $\widetilde{T} = \bigoplus_{i=0}^{a-1} \Omega^{-ni} T\langle i \rangle$
5. $B = \operatorname{End}_{\underline{\operatorname{gr}} \Lambda}(\widetilde{T})$

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A characterization

Theorem

The following statements are equivalent:

1. Λ is n-T-Koszul.
2. $\widetilde{T}$ is a tilting object in $\underline{\text{gr}}\,\Lambda$ and B is $(na-1)$-representation infinite.

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A finite dimensional algebra A is n-representation infinite if and only if ΔA is $(n+1)$-Koszul with respect to A.

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Proof of cor.: Assumptions OK for $\Lambda = \Delta A$ and $T = A$. Thm. yields

ΔA $(n+1)$-Koszul wrt. A $\iff$ $\begin{cases} A \text{ tilting obj. in } \underline{\text{gr}}\,\Delta A \\ A\ n\text{-rep. infinite} \end{cases}$ □

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A characterization

Corollary

There is a bijective correspondence

$$\left\{\begin{array}{l}\text{isomorphism classes}\\ \text{of } n\text{-representation}\\ \text{infinite algebras}\end{array}\right\} \rightleftarrows \left\{\begin{array}{l}\text{isomorphism classes of graded symmetric}\\ \text{finite dimensional algebras of highest}\\ \text{degree } 1 \text{ which are } (n+1)\text{-Koszul with}\\ \text{respect to their degree } 0 \text{ part}\end{array}\right\}.$$

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$$A \longmapsto \Delta A$$

$$\Lambda_0 \longleftarrow\!\!\mid \Lambda$$